Review Exam 1, MTH 205, Fall 2014

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QUESTION 1. Find the largest interval around x so that the LDE: $\frac{x-3}{\sqrt{3x+6}})y^{(4)}+(x-1)y = x^2+13, y'(1) = 7, y(1) = -6$ has a unique solution.

QUESTION 2. Solve for $x(t), y(t)$
$x'(t) - y(t) = 0$
$3x(t) + 2y'(t) = sin(t)$

QUESTION 3. 1) Show that $c_1 sin(x)+c_2 cos(x)$ is a solution to the LDE : $y^{(2)}+y = 0$, where c_1, c_2 are some constants.
2) Assume that $y'(\pi) = 1$ and $y(\pi/2) = 1$. Show that the given LDE has no solution in this case. Does this contradict the Initial Value Theorem?
3) Assume that $y'(\pi) = -1$ and $y(\pi/2) = 1$. Show that the given LDE has infinitely many solutions. Does this contradict the Initial Value Theorem?

QUESTION 4. 1) Find $\ell\{e^{4x}\}$,

2)$\ell^{-1}\{\frac{s+3}{s^2-7s+6}\}$.

3) Find $\ell\{5^{(2x+1)}\}$

4) Find $\ell^{-1}\{\frac{7-x}{(s+6)^4}\}$

QUESTION 5. Solve the following DE (use Laplace): 1) $y^{(2)} + 8y' + 12y = e^{-2x}$, $y(0) = 0, y'(0) = 0$.

2) $2y^{(2)} + 3y' + y = sin(2x)0$, $y(0) = 3$ and $y'(0) = -2$

QUESTION 6. (i) Find $\ell\{U(x-3)e^{2x}\}$

(ii) Find $\ell\{\int_0^x e^{-3r} sin(2r)\, dr\}$ [Hint : Note $e^{-3r} = e^{-3x}e^{3x-3r}$. Thus $\ell\{\int_0^x e^{-3r} sin(2r)\, dr\}$ = $\ell\{e^{-3x}\int_0^x e^{3x-3r} sin(2r)\, dr\}$]

(iii) Find $\ell^{-1}\{\frac{s+10}{(s+4)^4}\}$

(iv) Find $\ell^{-1}\{\frac{se^{-s}}{(s+3)^2+4}\}$

(v) Use CONVOLUTION Twice to find $\ell^{-1}\{\frac{1}{s^2(s^2+9)}\}$

(vi) Use convolution to find $\ell^{-1}\{\frac{1}{(s+4)^2((s+4)^2+9)}\}$ [Hint: Use 5]

(vii) Find $\ell^{-1}\{\frac{8e^{-3s}}{s^2-4}\}$

(viii) Let

$$k(x) = \begin{cases} 2 & 0 \le x < 4 \\ 6 & x \ge 4 \end{cases}$$

Solve $y^{(2)} - 6y' + 8y = k(x)$, y(0) = $y'(0) = 0$ [hint : first write K(x) in terms of Unit function]

(ix) Solve : $y'(x) - \int_0^x 2e^{x-r}y(r)\, dr = xe^x$, $y(0) = 0$

QUESTION 7. Use undetermined Coeff. Method to solve for $y(x)$:

(i) $y^{(2)} + 6y' - 7y = 0$

(ii) $y^{(6)} - 7y^{(5)} + 10y^{(4)} = 0$

(iii) $y^{(4)} + 6y^{(3)} + 9y^{(2)} = 0$

(iv) $y^{(6)} - 7y^{(5)} + 10y^{(4)} = 10$

(v) $y^{(2)} + 6y' - 7y = e^{-7x}$

(vi) $y^{(2)} + 2y' + 20y = 0$

(vii) $y^{(2)} + 2y' + 5y = 2x + 3$

(viii) $y^{(2)} + 6y' - 7y = cos(x)$

(ix) $y^{(2)} + 2y' + 5y = cos(x)$

(x) $y^{(2)} + 4y = sin(2x)$

QUESTION 8. Use undetermined Coeff. Method to solve for $y(x)$ for y_p you may use substitution or Laplace as in class:

(i) $y^{(2)} + 14y' + 49y = (3x^7 + 4x^6)e^{-7x}$ [for y_p here Laplace method take much less time than substitution / I think!, note that here if you want use substitution $y_p = x^2(a_7x^7 + a_6x^6 + ... + a_0)$]

(ii) $y^{(2)} + 9y' + 8y = 2x + 3$ [here y_p will take same time using either method, note $y_p = ax + b$ find a, b]

(iii) $y^{(4)} + y^{(2)} = cos(x)$ [note $y_h = c_1 + c_2x + c_3cos(x) + c_4sin(x)$. Since cos(x) appears only once in y_h and the given LDE = cos(x), we conclude $y_p = x(acos(x) + bsin(x))$, I guess substitution here is easier to find y_p.]

(iv) $y^{(2)} + 6y' + 25y = e^{-3x}$[Note $y_h = e^{-3x}(c_1cos(4x) + c_2sin(4x))$, for y_p note that e^{-3x} is not a solution to y_h. Here, $y_p = ae^{-3x}$, find a. Both method will take the same time.]

(v) $y^{(2)} + 4y = sin(3x)$[Hint $y_h = c_1cos(2x) + c_2sin(2x)$, and $y_p = acos(3x) + bsin(3x)$. I guess substitution is easier here!]

(vi) $y^{(2)} + 4y' + 4y = (x^2 + 3x - 2)e^{-2x}$. [for y_p, Laplace much easier!!!]

(vii) Solve $x^4y^{(2)} + 5x^3y' + 3x^2y = 0$.

(viii) Solve $3y^{(2)} + 6y' + 3 = e^{-x}U(x - 1)$. [Note here for y_p must use laplace, I have no idea for substitution!]

(ix) solve $y^{(2)} + y' - 6y = (x + 2)e^{2x}$

(x) solve $y^{(7)} + 7y^{(6)} + 10y^{(5)} = 32$

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